Math 110

Winter 2021

Lecture 9

Odds vs Probabilities

I tossed a coin 100 times. I landed 65 tails and 35 heads.

From this process

$P(\text{Tails}) = \frac{65}{100} = \frac{13}{20}$

\# tails   \# $\overline{\text{tails}}$

odds to land tails are 65 to 35

Notation for odds 65:35 $\Rightarrow$ 13:7

13 tails   7 $\overline{\text{tails}}$

\# times E happens

Odds for event E are $a : b$

\# times $\bar{E}$ happens

Use fraction, decimal, scientific notation for prob.

Use : for odds.

Standard deck of playing Cards

52 Cards, 26 Red, 12 Face, 4 Aces.

odds to draw

a) a red card — $26\text{ Red} : 26\ \overline{\text{Red}} \Rightarrow 1:1$

b) a Face Card — $12\text{ Face} : 40\ \overline{\text{Face}} \Rightarrow \boxed{3:10}$

c) an Ace — $4\text{ Aces} : 48\ \overline{\text{Aces}} \Rightarrow \boxed{1:12}$

d) a Face or Ace — $16:36 \Rightarrow \boxed{4:9}$

12 4

If odds for event E are $a:b$, then

$$P(E) = \frac{a}{a+b} \qquad P(\bar{E}) = \frac{b}{a+b}$$

ex: Given: odds for event E 22:3

$$P(E) = \frac{22}{22+3} = \boxed{\frac{22}{25}} \qquad P(\bar{E}) = \frac{3}{22+3} = \boxed{\frac{3}{25}}$$

Suppose 50 shots in a basketball were randomly selected, and 38 were made, and 12 were missed.

odds to make a shot $38:12 \Rightarrow 19:6$

odds to miss a shot $12:38 \Rightarrow 6:19$

$$P(\text{Make}) = \frac{19}{19+6} = \boxed{\frac{19}{25}} \qquad P(\overline{\text{Make}}) = \frac{6}{19+6} = \boxed{\frac{6}{25}}$$

If $P(E)$ is given, then

odds for event E are $\frac{P(E)}{P(\bar{E})}$, Reduce, write using : Notation

$P(\text{passing this class}) = .72$

25 students
18 pass
7 $\overline{\text{pass}}$

$P(\text{pass}) = .72$ $P(\overline{\text{pass}}) = .28$

odds for passing $\frac{.72}{.28} \Rightarrow \boxed{18:7}$

$.72 \div .28$ Math 1: Enter $\frac{18}{7}$

Prob. that LA Lakers win the championship this year is 80%.

$P(W) = .8$, $P(\overline{W}) = .2$

odds for Winning are $\frac{.8}{.2} \Rightarrow \boxed{4:1}$

$= 4 = \frac{4}{1}$

Must be Part of answer.

4 : 1

bet \$4 Net Profit \$1

a : b

bet Net return

If you bet against them

1 : 4

\$1 bet \$4 Net return

A certain game has the following odds:

4 : 21

How much do you need to bet in order to have net return of $630?

$$\frac{\$4 \text{ bet}}{\$21 \text{ Net}} \qquad \frac{\$x \text{ bet}}{\$630 \text{ Net}}$$

Solve $\frac{4}{21} = \frac{x}{630}$

Cross-Multiply

$21x = 4(630)$

$x = \frac{4(630)}{21}$

$x = 120$

$120

Counting:

If You toss a Coin once ⇒ T or H   2 outcomes

If You toss a Coin twice ⇒ TT   TH   HT   HH

$\underline{2} \cdot \underline{2} = \boxed{4}$

Toss a Coin Once,

If heads, toss it again

If tails, Roll a 6-sided die.

HH   HT   T1   T2   T3   T4   T5   T6

8 outcomes

Password for ATM Card.

Choose 4 digits $\{0, 1, 2, \dots, 9\}$

1) Repetition $\underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = \boxed{10{,}000}$

2) No Repetition $\underline{10}\ \underline{9}\ \underline{8}\ \underline{7} = \boxed{5040}$

You need to choose a password that starts with a letter, follows by 3 digits, then another letter.

Letters are Case Sensitive, digits cannot be repeated

How many passwords?

$$\underline{52} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{52}$$

$$= \boxed{1{,}946{,}880}$$

There are 5 people: Allen, Bill, Carol, Diane, Edward

I need to select 2 of them

~~AB~~	~~AC~~	~~AD~~	~~AE~~
BA	~~BC~~	~~BD~~	~~BE~~
CA	CB	~~CD~~	~~CE~~
DA	DB	DC	~~DE~~
EA	EB	EC	ED

$\underline{5}\ \underline{4} = 20$ Selections

10 ways

Suppose order does <u>not</u> matter

${}_nC_r$

items select

order does not matter

${}_5C_2 = \boxed{10}$

5 [Math] PRB [$_nC_r$] 2 [Enter]

12 students in a first-grade class are part of the basketball team. 5 need to start the game. How many ways can this be done?

$${}_{12}C_5 = 792$$

CA Lotto (NO Mega #)

50 numbers, choose any 5 numbers

How many total # of selections? ${}_{50}C_5 = \boxed{2{,}118{,}760}$

A standard deck of playing cards.

Draw 3 cards, keep your cards, order does not matter.

1) How many ways can this be done?

$${}_{52}C_3 = 22100$$

2) How many ways can you draw 3 face cards?

$${}_{12}C_3 = 220$$

3) $P(\underbrace{\text{draw 3 face cards}}_{\text{Rare event}}) = \dfrac{220}{22100} = .010$ (.00995) $= \boxed{\dfrac{11}{1105}}$

$0 <$ $\leq .05$

Draw 2 Cards, No replacement, order does not matter.

1) How many ways can this be done?

$_{52}C_2 = 1326$

2) How many ways can we get 2 aces?

$_4C_2 = 6$

3) $P(\text{Draw 2 Aces}) = \dfrac{_4C_2}{_{52}C_2} = \dfrac{6}{1326} = \boxed{\dfrac{1}{221}}$

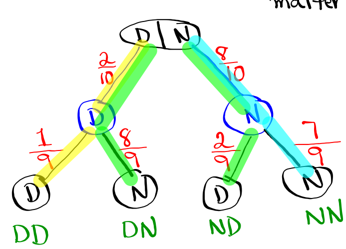
4) Find odds to draw 2 Aces.

1 : 220

odds against ⇒ 220 : 1

A box contains 2 Dimes & 8 Nickels.

Draw 2 Coins, No replacement, order does not matter.



$P(20¢) = P(DD) = \frac{2}{10} \cdot \frac{1}{9} = \boxed{\frac{2}{90}}$

$P(15¢) = P(1D\,1N) = P(DN \text{ or } ND) = \frac{2}{10} \cdot \frac{8}{9} + \frac{8}{10} \cdot \frac{2}{9}$

$= \boxed{\frac{32}{90}}$

$P(10¢) = P(NN) = \frac{8}{10} \cdot \frac{7}{9} = \boxed{\frac{56}{90}}$

Total ¢	P(Total ¢)
20¢	2/90
15¢	32/90
10¢	56/90

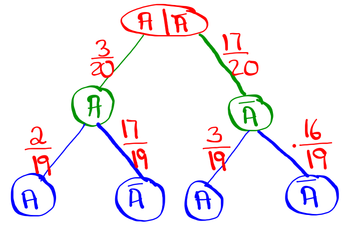
Total ¢ → L1

P(Total ¢) → L2

Use L1 & L2 to find $\bar{x}$ & n

$\bar{x} = 12$, n = 1

A deck of playing cards has 20 cards, and 3 Aces.
Draw 2 cards, No replacement.



$P(2 \text{ Aces}) = \frac{3}{20} \cdot \frac{2}{19} = \boxed{\frac{6}{380}}$

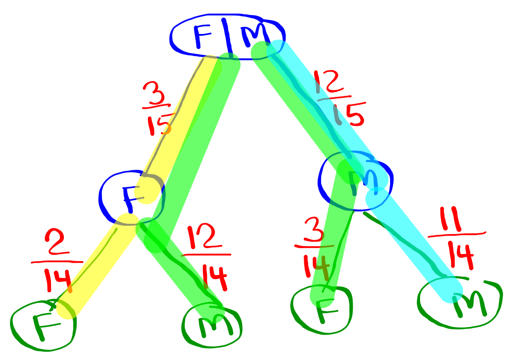
$P(1 \text{ Ace only}) = \frac{3}{20} \cdot \frac{17}{19} + \frac{17}{20} \cdot \frac{3}{19} = \boxed{\frac{102}{380}}$

$P(\text{NO Aces}) = \frac{17}{20} \cdot \frac{16}{19} = \boxed{\frac{272}{380}}$

# Aces	P(# Aces)
0	272/380
1	102/380
2	6/380

Clear all lists
Aces → L1, P(#Aces) → L2
Use L1 & L2 to find
$\bar{x} = .3$ S = blank   n = 1

3 Females, 12 Males, Select 2 people, order does not matter.



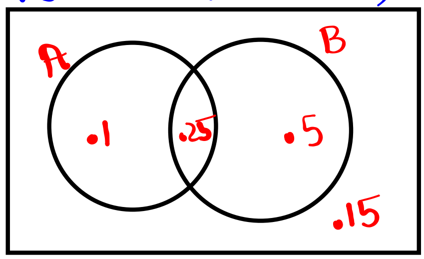
$P(2 \text{ Females}) = \frac{3}{15} \cdot \frac{2}{14} = \boxed{\frac{6}{210}}$

$P(1 \text{ Female}) = \frac{3}{15} \cdot \frac{12}{14} + \frac{12}{15} \cdot \frac{3}{14} = \boxed{\frac{72}{210}}$

$P(\text{NO Females}) = \frac{12}{15} \cdot \frac{11}{14} = \boxed{\frac{132}{210}}$

$P(A) = .35$, $P(B) = .75$ $P(A \text{ and } B) = .25$

1) Draw Venn Diagram



2) $P(A \text{ or } B)$

$= .1 + .25 + .5 = \boxed{.85}$

3) $P(A|B)$

$= \dfrac{P(A \text{ and } B)}{P(B)} = \dfrac{.25}{.75}$

$= \boxed{\dfrac{1}{3}} = \boxed{.333}$

$P(A) = .3$ $P(B) = .4$, A & B are independent events

1) $P(\overline{A}) = 1 - .3$

$= \boxed{.7}$

2) $P(A \text{ and } B) = P(A) \cdot P(B)$

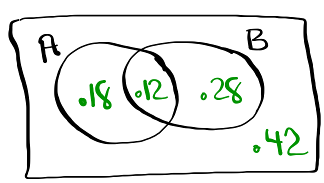
$= \boxed{.12}$

3) $P(A \text{ or } B)$

$= P(A) + P(B) - P(A \text{ and } B)$

$= .3 + .4 - .12 = \boxed{.58}$

4) Venn Diagram



5) $P(A \text{ only or } B \text{ only}) = .18 + .28 = \boxed{.46}$

Please work on SG 13 & 14.